Constrained Maximization of Lattice Submodular Functions

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Motivation

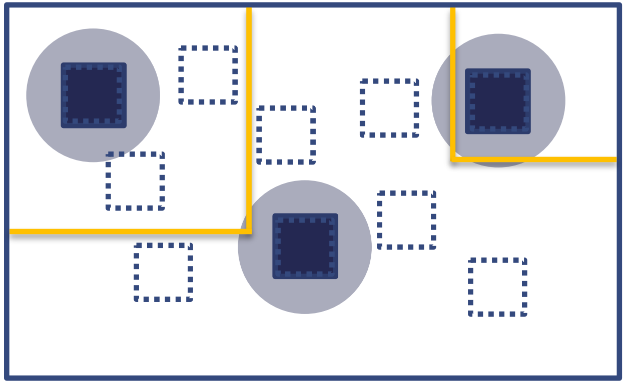
Submodular functions

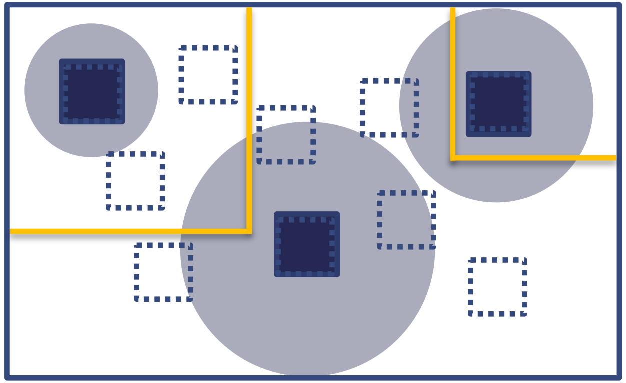
They model "diversity", "coherence" of sets:

- Coverage functions
- Log-determinant
- Graph cuts and many more

Constrained maximization of ~~Set~~ Lattice Submodular functions

$$\begin{bmatrix} 0 \\ 1 \\ \hline \vdots \\ \hline 1 \end{bmatrix} \begin{matrix} \leq 1 \\ \leq 1 \\ \leq 1 \end{matrix}$$





$$\begin{bmatrix} 0 \\ 3 \\ \hline \vdots \\ \hline 2 \end{bmatrix} \begin{matrix} \leq 3 \\ \leq 1 \\ \leq 2 \end{matrix}$$

Overview

Lattice Submodular

$$\underset{\mathbf{x}}{\text{maximize}} \quad f(\mathbf{x})$$
$$\text{subject to} \quad \mathbf{x} \in \mathcal{M}$$

Discrete Polymatroid

Continuous relaxation using the Generalized Multilinear Extension

Rounding back with a loss which depends on the "distance" to DR-submodularity

DR-submodular

$$\underset{\boldsymbol{\rho}}{\text{maximize}} \quad F(\boldsymbol{\rho})$$
$$\text{subject to} \quad \boldsymbol{\rho} \in \Delta_n^{k-1}, T(\boldsymbol{\rho}) \in \mathcal{P}(\mathcal{M})$$

Probability Simplex

Integral Polymatroid

$$f(\mathbf{x}_{\mathrm{I}}^*) \geq (1 - \tfrac{1}{e}) \max_{\mathbf{x} \in \mathcal{M}} f(\mathbf{x}) - \lambda \left(\mathbb{E}_{\boldsymbol{\rho}*} \|\mathbf{x}\|^2 - \|\mathbf{x}_{\mathrm{I}}^*\|^2 \right)$$